Two-loop renormalization of $N = 1$ supersymmetric electrodynamics, regularized by higher derivatives.

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Abstract

Two-loop β-function and anomalous dimension are calculated for $N = 1$ supersymmetric quantum electrodynamics, regularized by higher derivatives. The result for two-loop contribution to β-function appears to be equal to 0, does not depend on the form of regularizing term and does not lead to anomaly puzzle. Two-loop anomalous dimension can be also made independent on parameters of the chosen regularization by a special choice of subtraction scheme.

1 Introduction.

Investigation of quantum corrections in supersymmetric theories is a very important and complicated problem. In principle, supersymmetric theories have better ultraviolet behavior, than nonsupersymmetric models. For example, in $N = 2$ Yang-Mills theory perturbative divergences are present only in one-loop diagrams. In principle it follows from the fact, that in supersymmetric theories axial anomaly and anomaly of energy-momentum tensor trace belong to the same supermultiplet [1, 2, 3, 4]. The axial anomaly is known to be completely defined by the one-loop approximation [5, 6], while the trace anomaly is proportional to β-function [7]. Therefore, due to the supersymmetry the β-function should be also defined by the one-loop approximation. The same arguments can be applied to $N = 1$ supersymmetric theories. However, explicit calculations of radiative corrections show, that the β-function in $N = 1$ supersymmetric models has contributions from higher loops [8, 9, 10, 11]. This contradiction is

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usually called "anomaly puzzle" and was investigated in a large number of papers, for example [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23].

Usually different proposals to solve anomaly puzzle require to fix the form of the β-function in all orders of perturbation theory. For example, in theories with matter the β-function should be related with the anomalous dimension. For the first time such β-function was obtained by Novikov, Shifman, Vainshtein and Zakharov (NSVZ) from investigation of the structure of instanton corrections [24]. Later, this result was checked by explicit calculations, which were usually made by the dimensional reduction technique [25]. Two-loop β-function, obtained in this regularization, is shown to coincide with a prediction, following from NSVZ exact expression. However, three-loop β-function [26, 27, 28] does not agree with it. Nevertheless, the deviations can be removed by a redefinition of the coupling constant [29], the possibility of such redefinition being highly nontrivial [30]. In principle it is possible to relate $\overline{\mathrm{DR}}$ scheme and NSVZ scheme order by order [31] in the perturbation theory.

It is necessary to especially mention paper [23], in which β-function is shown to depend on the normalization of matter and gauge superfields. In particular, NSVZ β-function can be obtained after a special rescaling of these superfield, which reduces kinetic terms to the canonical renormalization. Otherwise, (without rescalings) β-function is argued to be completely defined by the one-loop approximation. So, it is really quite possible to obtain zero contributions of higher loops. The problem is how to calculate the corrections. For example, it is possible to look for the regularization, in which β-function is equal to 0 or coincides with NSVZ exact β-function. From this point of view the most attractive regularization for supersymmetric theories is the regularization by higher covariant derivatives [32, 33]. For the supersymmetric Yang-Mills theory the Lagrangian of the regularized theory was constructed in [34]. For electrodynamics construction of the regularized Lagrangian is simpler, because instead of covariant derivatives it is necessary to use usual derivatives. However, calculation of diagrams, regularized by higher covariant derivatives is rather complicated. In particular, explicit calculation of the one-loop quantum correction [1] for the (nonsupersymmetric) Yang-Mills theory was made rather recently [35, 36] and gives the same result as the dimensional regularization. In principle it is possible to prove, that one-loop calculations using higher covariant derivative regularization (certainly, complemented by the additional regularization for one-loop diagrams) always gives the same result as the dimensional regularization [37]. Investigation of two-loop corrections in theories, regularized by higher derivatives has not yet been done.

In this paper we try to understand features of higher derivative regularization in supersymmetric theories and calculate two-loop renormgroup functions for massless $N=1$ supersymmetric quantum electrodynamics in this regularization.

The paper is organized as follows:

In Section 2 we introduce notations and remind some information about $N=$

[1]Note, that introducing of a term with higher covariant derivatives does not lead to regularization of one-loop divergences. For the one-loop divergences it is necessary to use one more regularization, for example, introduce Pauli-Villars fields.

1 supersymmetric electrodynamics. In the next Section 3 the considered model is regularized by higher derivatives. Two-loop β-function and anomalous dimension are calculated in Section 4. Agreement of the results with renormgroup equations is checked in Section 5. A brief summary and discussion is presented in Conclusion. Technical details of calculations can be found in the Appendix.

2 Supersymmetric quantum electrodynamics.

$N = 1$ supersymmetric massless electrodynamics in the superspace is described by the following action:

$$S_0 = \frac{1}{4e_0^2}\mbox{Re}\int d^4x\, d^2\theta\, W_a C^{ab} W_b + \frac{1}{4}\int d^4x\, d^4\theta\, \Big(\phi^* e^{2V}\phi + \tilde\phi^* e^{-2V}\tilde\phi\Big). \qquad (1)$$

Here ϕ and $\tilde\phi$ are chiral superfields, which in components can be written as

$$\begin{aligned}
&\phi(y,\theta) = \varphi(y) + \bar\theta(1+\gamma_5)\psi(y) + \frac{1}{2}\bar\theta(1+\gamma_5)\theta f(y);\\
&\tilde\phi(y,\theta) = \tilde\varphi(y) + \bar\theta(1+\gamma_5)\tilde\psi(y) + \frac{1}{2}\bar\theta(1+\gamma_5)\theta \tilde f(y),
\end{aligned} \qquad (2)$$

where $y^\mu = x^\mu + i\bar\theta\gamma^\mu\gamma_5\theta/2$ are chiral coordinates, φ and $\tilde\varphi$ are complex scalar fields, ψ and $\tilde\psi$ are Maiorana spinors, which can be unified in a Dirac spinor

$$\Psi = \frac{1}{\sqrt{2}}\Big((1+\gamma_5)\psi + (1-\gamma_5)\tilde\psi\Big), \qquad (3)$$

and f and $\tilde f$ are auxiliary complex scalar fields.

V is a real abelian superfield, which is a supersymmetric generalization of the gauge field. In the Wess-Zumino gauge this superfield can be written as

$$V(x,\theta) = \frac{1}{2}\bar\theta\gamma^\mu\gamma_5\theta\, A_\mu(x) + i\sqrt{2}(\bar\theta\theta)(\bar\theta\gamma_5\chi(x)) + \frac{1}{4}(\bar\theta\theta)^2 D(x), \qquad (4)$$

where A_μ is a gauge field, χ is a Maiorana spinor and D is a real auxiliary scalar field.

The superfield W_a is a supersymmetric generalization of the field strength tensor and in the abelian case is defined as

$$W_a = \frac{1}{16}\bar D(1-\gamma_5)D\Big[(1+\gamma_5)D_a V\Big], \qquad (5)$$

where the supersymmetric covariant derivative D is written as

$$D = \frac{\partial}{\partial\bar\theta} - i\gamma^\mu\theta\,\partial_\mu. \qquad (6)$$

W_a is a chiral Weil spinor and in components is written as

$$W_a = \frac{1}{2}(1+\gamma_5)\Big(\sqrt{2}\,\chi(y) - i\theta D(y) + \frac{1}{2}\Sigma_{\mu\nu}\theta F^{\mu\nu} - \frac{i}{\sqrt{2}}\gamma^\mu \mathcal{D}_\mu \chi(y)\,\bar\theta(1+\gamma_5)\theta\Big). \qquad (7)$$

Using expansions (2), (4) and (7) it is easy to see, that in components action (1) takes the following form:

$$S = \int d^4x \left(-\frac{1}{4e_0^2}F_{\mu\nu}^2 + \frac{i}{e_0^2}\bar\chi\gamma^\mu\partial_\mu\chi + \frac{1}{2e_0^2}D^2 + \right.$$
$$\left. + |\mathcal{D}_\mu\varphi|^2 + |\mathcal{D}_\mu\tilde\varphi|^2 + i\bar\Psi\gamma^\mu\mathcal{D}_\mu\Psi + |f|^2 + |\tilde f|^2 \right). \qquad (8)$$

3 Higher derivative regularization of $N = 1$ supersymmetric electrodynamics.

To regularize model (1) by higher derivatives let us modify its action by the follwing way:

$$S_0 \to S = S_0 + S_\Lambda =$$
$$= \frac{1}{4e_0^2}\mathrm{Re}\int d^4x\, d^2\theta\, W_a C^{ab}\Big(1 + \frac{\partial^{2n}}{\Lambda^{2n}}\Big)W_b +$$
$$+ \frac{1}{4}\int d^4x\, d^4\theta\,\Big(\phi^* e^{2V}\phi + \tilde\phi^* e^{-2V}\tilde\phi\Big). \qquad (9)$$

Note, that the considered model is abelian and the superfield W^a is gauge invariant. Therefore, a regularizing term should contain usual derivatives instead of the covariant ones.

It is easy to see, that the degree of divergence for model (9) is equal to

$$\omega_\Lambda = \begin{cases} 2 - 2n(L-1) - E_\phi(n+1), & E_\phi \ne 0; \\ -2n(L-1), & E_\phi = 0, \end{cases} \qquad (10)$$

where L is a number of loops, E_ϕ is a number of external ϕ-lines. A number of external V-lines is denoted by E_V.

From equation (10) we conclude, that divergences are present only in one-loop diagrams if $n \ge 2$. Therefore, the one-loop diagrams should be considered separately. In particular to regularize them it is necessary to use anticommuting Pauli-Villars fields [6] Φ and $\tilde\Phi$ with the Lagrangian

$$S_{PV} = \frac{1}{4}\int d^4x\, d^4\theta\, \Big(\Phi^* e^{2V}\Phi + \tilde\Phi^* e^{-2V}\tilde\Phi\Big) + \\ + \frac{1}{2}\int d^4x\, d^2\theta\, M\tilde\Phi\Phi + \frac{1}{2}\int d^4x\, d^2\bar\theta\, M\tilde\Phi^*\Phi^*. \qquad (11)$$

Below we will assume, that $M = a\Lambda$, where a is a constant.

4 Calculation of two-loop renormgroup functions.

Let us calculate two-loop β-function and anomalous dimension for a model, described by action (9).

Its quantization is made similar to the case, considered in [38], and is not discussed here. It should be only mentioned, that the gauge invariance is fixed by addition of the following terms:

$$S_{gf} = -\frac{1}{64e_0^2}\int d^4x\, d^4\theta\, \left(V D^2\bar D^2\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)V + V\bar D^2 D^2\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)V\right), \qquad (12)$$

where

$$D^2 \equiv \frac{1}{2}\bar D(1+\gamma_5)D; \qquad \bar D^2 \equiv \frac{1}{2}\bar D(1-\gamma_5)D. \qquad (13)$$

After adding of such gauge fixing, kinetic term for the gauge field can be written in the most simple form:

$$S_{gauge} + S_{gf} = \frac{1}{4e_0^2}\int d^4x\, d^4\theta V\partial^2\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)V. \qquad (14)$$

Note, that the considered case corresponds to the gauge group $U(1)$ and, therefore, diagrams with ghost loops are absent.

Due to the supersymmetric gauge invariance

$$V \to V - \frac{1}{2}(A + A^+); \qquad \phi \to e^A\phi; \qquad \tilde\phi \to e^{-A}\tilde\phi, \qquad (15)$$

where A is an arbitrary chiral superfield, the renormalized action of the considered model can be written as

$$S_{ren} = \frac{1}{4e^2(\Lambda/\mu)}\mathrm{Re}\int d^4x\, d^2\theta\, W_a C^{ab}\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)W_b + \\ + \frac{1}{4}Z(\Lambda/\mu)\int d^4x\, d^4\theta\, \Big(\phi^* e^{2V}\phi + \tilde\phi^* e^{-2V}\tilde\phi + \Phi^* e^{2V}\Phi + \tilde\Phi^* e^{-2V}\tilde\Phi\Big) + \\ + \frac{1}{2}\int d^4x\, d^2\theta\, M\tilde\Phi\Phi + \frac{1}{2}\int d^4x\, d^2\bar\theta\, M\tilde\Phi^*\Phi^*. \qquad (16)$$

Here a mass of Pauli-Villars fields is assumed to be proportional to the constant Λ and the Lagrangian of Pauli-Villars fields is chosen so that contributions of these fields cancel all divergences of the Feinman graphs, remaining after introducing of higher derivative term.

In our notations β-function and anomalous dimension are defined as

$$\beta = \frac{d}{d\ln\mu}\left(\frac{e^2}{4\pi}\right); \qquad \gamma = -\frac{d\ln Z}{d\ln\mu}. \tag{17}$$

In the two-loop approximation β-function can be found after calculation of diagrams with $E_V = 2$, $E_\phi = 0$, presented at Figure 1. Similarly, anomalous dimension can be found after calculation of diagrams with $E_V = 0$, $E_\phi = 0$, presented at Figure 2.

Having performed the calculations, after Wick rotation in the Euclidean space we obtained the following results:

1. Two-loop contribution to the effective action, corresponding to the two-point Green function of the gauge field, is written as

$$\begin{aligned}\Delta\Gamma^{(2)}_V &= \int d^2\theta\,\frac{d^4p}{(2\pi)^4} W_a(p) C^{ab} W_b(-p)\times\\
&\times\Bigg\{-\int\frac{d^4k}{(2\pi)^4}\frac{1}{2k^2(k+p)^2}+\int\frac{d^4k}{(2\pi)^4}\frac{1}{2(k^2+M^2)\Big((k+p)^2+M^2\Big)}+\\
&+e_0^2\int\frac{d^4k}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{(k+p+q)^2+q^2-k^2-p^2}{\Big(k^2+k^{2n+2}/\Lambda^{2n}\Big)(k+q)^2(k+p+q)^2q^2(q+p)^2}-\\
&-\frac{e_0^2}{2\pi^2}\ln\frac{\Lambda}{\mu}\lim_{M\to\infty}\int\frac{d^4k}{(2\pi)^4}\frac{M^2}{(k^2+M^2)^2\Big((k+p)^2+M^2\Big)}\Bigg\}.\end{aligned} \tag{18}$$

where $M = a\Lambda$ is a mass of Pauli-Villars fields.

It is important to note, that due to Konishi anomaly [39] a sum of diagrams with insertion of one-loop counterterms appears to be different from 0. In the considered model expressions for these diagrams are well defined due to existence of Pauli-Villars fields. They were calculated according to a method, described in [40]. The result appeared to be proportional to contribution of Pauli-Villars fields at $M\to\infty$ and was not equal to 0.

2. Two-loop contribution to the effective action, corresponding to the two-point function of the matter superfield, is written as

$$\begin{aligned}\Delta\Gamma^{(2)}_\phi &= \int d^4\theta\,\frac{d^4p}{(2\pi)^4}\Big(\phi^*(p)\,\phi(-p)+\tilde\phi^*(p)\,\tilde\phi(-p)\Big)\times\\
&\times\Bigg\{\int\frac{d^4k}{(2\pi)^4}\frac{e_0^2}{2k^2(k+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)}+\end{aligned}$$

$$
+\int\frac{d^4k}{(2\pi)^4}\,\frac{d^4q}{(2\pi)^4}\,\frac{e_0^4}{k^2q^2(k+p)^2(q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)}+
$$

$$
+\int\frac{d^4k}{(2\pi)^4}\,\frac{d^4q}{(2\pi)^4}\,\frac{e_0^4}{k^2q^2(q+p)^2(k+q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)}-
$$

$$
-\frac{e_0^4}{4\pi^2}\ln\frac{\Lambda}{\mu}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2(k+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)}-\int\frac{d^4k}{(2\pi)^4}\,\frac{d^4q}{(2\pi)^4}\,\times
$$

$$
\times\frac{e_0^4(k+q+2p)^2}{k^2(k+p)^2q^2(q+p)^2(k+q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)}+
$$

$$
+\int\frac{d^4q}{(2\pi)^4}\,\frac{e_0^4}{q^2(q+p)^2\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}\Bigg(\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2(k+q)^2}-
$$

$$
-\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{(k^2+M^2)\Big((k+q)^2+M^2\Big)}\Bigg). \qquad (19)
$$

Results (18) and (19) are evidently invariant under supersymmetry transformations, because they can be written as integrals from products of superfields (2) and (7) over the superspace. The gauge invariance is absent in equation (19) because the calculations were made only for diagrams with $E_V=0$, $E_\phi=2$. Adding of terms, corresponding to diagrams with arbitrary E_V and $E_\phi=2$, will certainly restore the gauge invariance.

The integrals in equations (18) and (19) are calculated in Appendix A. Using results, obtained there, it is easy to find, that counterterms, needed to cancel two-loop divergences can be written as

$$
\Delta S=\frac{1}{16\pi^2}\ln\frac{\Lambda}{\mu}\,\mathrm{Re}\int d^4x\,d^2\theta\;W_aC^{ab}\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)W_b+
$$

$$
+\frac{1}{4}\int d^4x\,d^4\theta\,\Big(\phi^*e^{2V}\phi+\tilde\phi^*e^{-2V}\tilde\phi\Big)\times
$$

$$
\times\Bigg\{-\frac{\alpha_0}{\pi}\ln\frac{\Lambda}{\mu}+\frac{\alpha_0^2}{\pi^2}\ln^2\frac{\Lambda}{\mu}-\frac{\alpha_0^2}{\pi^2}\ln\frac{\Lambda}{\mu}\Bigg(\ln\frac{M}{\Lambda}+\frac{3}{2}\Bigg)\Bigg\}, \qquad (20)
$$

that corresponds to

$$
\frac{4\pi^2}{e^2}=\frac{\pi}{\alpha_0}+\ln\frac{\Lambda}{\mu}+O(\alpha_0^2); \qquad (21)
$$

$$
Z\Big(\Lambda/\mu\Big)=1-\frac{\alpha_0}{\pi}\,\ln\frac{\Lambda}{\mu}+\frac{\alpha_0^2}{\pi^2}\,\ln^2\frac{\Lambda}{\mu}-\frac{\alpha_0^2}{\pi^2}\,\ln\frac{\Lambda}{\mu}\Bigg(\ln\frac{M}{\Lambda}+\frac{3}{2}\Bigg)+O(\alpha_0^3), \qquad (22)
$$

Therefore the two-loop β-function and anomalous dimension of $N=1$ supersymmetric quantum electrodynamics, regularized by higher derivatives, are written as

$$\beta = \frac{\alpha^2}{\pi} + O(\alpha^4);$$
$$\gamma(\alpha) = -\frac{\alpha}{\pi} - \frac{\alpha^2}{\pi^2}\Big(\ln\frac{M}{\Lambda} + \frac{3}{2}\Big) + O(\alpha^3). \tag{23}$$

In particular, two-loop contribution to the β-function appears to be 0, so that the beta function is completely defined by the one-loop approximation.

The anomalous dimension $\gamma(\alpha)$ in the two-loop approximation does not depend on n or, by other words, on a form of regularizing term. However, it depends on the ratio of Pauli-Villars mass to the constant Λ. Nevertheless, the dependence on M/Λ can be removed by addition of finite counterterms, proportional to $\ln M/\Lambda$:

$$S_{ren} = \frac{1}{4e^2(\Lambda/\mu)}\mbox{Re}\int d^4x\,d^2\theta\,W_a C^{ab}\Big(1+\frac{\partial^{2n}}{\Lambda^{2n}}\Big)W_b + \frac{1}{16\pi^2}\ln\frac{M}{\Lambda}\times$$
$$\times\,\mbox{Re}\int d^4x\,d^2\theta\,W_a C^{ab} W_b + Z(\Lambda/\mu)\frac{1}{4}\int d^4x\,d^4\theta\,\Big(\phi^* e^{2V}\phi + \tilde\phi^* e^{-2V}\tilde\phi\Big), \tag{24}$$

that corresponds to another subtraction scheme, in which anomalous dimension is equal to

$$\gamma(\alpha) = -\frac{\alpha}{\pi} - \frac{3\alpha^2}{2\pi^2} + O(\alpha^3) \tag{25}$$

and does not depend on both n and M/Λ. In principle, in this scheme it is possible to consider, that $\Lambda\to\infty$, $M/\Lambda\to\infty$ instead of $M = a\Lambda$.

5 Comparing of the results with predictions of the renormalization group method.

The obtained results can be checked by the renormalization group method. It is well known [41], that in renormalizable theories terms, proportional to $\ln^2\mu/\Lambda$ are completely defined by one-loop counterterms. Therefore, it is possible to calculate such terms by renormgroup equations and compare them with the result of calculation of Feinman graphs.

Using the notation

$$t = \ln\frac{\mu}{\Lambda} \tag{26}$$

for the considered model renormgroup equations can be written as

$$Z(t) = \exp\Big\{-\int dt\,\gamma\Big(\alpha(t)\Big)\Big\};\qquad t = \int\frac{d\alpha}{\beta(\alpha)}. \tag{27}$$

Because in the one-loop approximation the β-function is equal to

$$\beta(\alpha) = \alpha^2\beta_1 + O(\alpha^3), \tag{28}$$

in the lowest order

$$\alpha(t) = \alpha_0\Big(1 + \beta_1\alpha_0\, t + O(\alpha_0^2)\Big), \tag{29}$$

where $\alpha_0 = \alpha(0)$. Expanding the anomalous dimension in powers of α

$$\gamma(\alpha) = \alpha\gamma_1 + \alpha^2\gamma_2 + O(\alpha^3) = \gamma_1\Big(\alpha_0 + \beta_1\alpha_0^2\, t\Big) + \alpha_0^2\,\gamma_2 + O(\alpha_0^3) \tag{30}$$

and substituting it to the first equation of (27), we obtain, that

$$Z(t) = 1 - \gamma_1\alpha_0\, t - \gamma_1\beta_1\alpha_0^2\, t^2/2 - \gamma_2\alpha_0^2\, t + \gamma_1^2\alpha_0^2\, t^2/2 + O(\alpha_0^3). \tag{31}$$

Taking into account, that according to the results of one-loop calculations $\gamma_1 = -1/\pi$ and $\beta_1 = 1/\pi$, the function Z should take the following form:

$$Z(\Lambda/\mu) = 1 - \frac{\alpha_0}{\pi}\,\ln\frac{\Lambda}{\mu} + \frac{\alpha_0^2}{\pi^2}\,\ln^2\frac{\Lambda}{\mu} + \gamma_2\alpha_0^2\,\ln\frac{\Lambda}{\mu} + O(\alpha_0^3). \tag{32}$$

Comparing this expression with equation (22) we see, that terms proportional to $\ln^2\mu/\Lambda$ coincide, that can be considered as a check of performed calculations.

6 Conclusion.

In this paper we calculated two-loop β-function and anomalous dimension for $N = 1$ supersymmetric massless quantum electrodynamics, regularized by higher derivatives. In particular, two-loop contribution to the β-function is found to be 0 and not to depend on the form of higher derivative term. As we mentioned above, this result follows from the fact, that the axial anomaly and the anomaly of energy-momentum tensor trace in the considered model belongs to the one supermultiplet. However, to obtain it we have to perform calculations using higher covariant derivative regularization. In principle, this regularization (complemented by the Pauli-Villars regularization for one-loop diagrams) allows to perform easy calculation of diagrams with insertion of counterterms, which are proportional to Konishi anomaly [39] and have nonzero contribution. Possibly the results of the paper allow to assume, that contributions of all higher loops in the β-function of $N = 1$ supersymmetric electrodynamics, regularized by higher derivatives, are also equal to 0. However, to be completely sure in it, it is necessary to calculate scheme dependent three-loop β-function.

Two-loop anomalous dimension is found to be independent on the form of higher derivative term if one renormalizes the coupling constant in this term. However, $\gamma(\alpha)$ depends on the ratio of Pauli-Villars mass to the constant Λ. Nevertheless, it is possible to make anomalous dimension completely independent on parameters of higher derivative regularization (n and M/Λ) if one introduces some finite counterterms in the renormalized action, that actually corresponds to a different choice of renormalization scheme.

To check the results of calculations we verified, that terms, proportional to $\ln^2 \mu/\Lambda$, obtained after calculation of Feinman graphs agree with predictions of renormgroup equations.

Note, that the result for β-function does not contradict to NSVZ results, because in considered model β-function depends on the normalization of the matter superfields [23]. In particular, after a scale transformation making matter superfields canonically normalized, it is possible to obtain NVSZ β-function. However, in the present paper this statement was not checked by explicit calculations and we hope to make it later.

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Appendix.

A Calculation of integrals, regularized by higher derivatives.

Let us calculate divergent parts of integrals, which were encountered in equations (19) and (18). (Remember, that they are written in Euclidean space, Wick rotation having been made.)

$$I_1 = \int d^4k \frac{1}{k^2(k+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)};$$

$$I_2 = \int d^4k\, \frac{1}{k^2(k+p)^2} - \int d^4k\, \frac{1}{(k^2+M^2)\Big((k+p)^2+M^2\Big)};$$

$$I_3 = \int d^4k\, d^4q \frac{(k+p+q)^2+q^2-k^2-p^2}{k^2\Big(1+k^{2n}/\Lambda^{2n}\Big)(k+q)^2(k+p+q)^2q^2(q+p)^2};$$

$$I_4 = \lim_{M\to\infty}\int d^4k\, \frac{M^2}{(k^2+M^2)^2\Big((k+p)^2+M^2\Big)};$$

$$I_5 = \int d^4k\, d^4q\, \frac{1}{k^2(k+p)^2q^2(q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)} = I_1^2;$$

$$I_6 = \int d^4k\, d^4q\, \frac{(k+q+2p)^2}{k^2(k+p)^2q^2(q+p)^2(k+q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)};$$
$$I_7 = \int d^4k\, d^4q\, \frac{1}{k^2q^2(q+p)^2(k+q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)};$$
$$I_8 = \int d^4q\, \frac{1}{q^2(q+p)^2\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}\, I_2(q/M). \qquad (33)$$

In order to calculate integral I_1 it is possible to use four-dimensional spherical coordinates

$$\begin{aligned}
k_1 &= k\sin\theta_3\sin\theta_2\sin\theta_1;\\
k_2 &= k\sin\theta_3\sin\theta_2\cos\theta_1;\\
k_3 &= k\sin\theta_3\cos\theta_2;\\
k_4 &= k\cos\theta_3.
\end{aligned} \qquad (34)$$

and direct fourth axis along p^μ, so that the integrand will depend only on θ_3 and

$$\int d^4k = \int\limits_0^\infty k^3dk \int\limits_0^\pi d\theta_3\, \sin^2\theta_3 \int\limits_0^\pi d\theta_2\, \sin\theta_2 \int\limits_0^{2\pi} d\theta_1 = 4\pi \int\limits_0^\infty k^3dk \int\limits_0^\pi d\theta_3\, \sin^2\theta_3 =$$
$$= \Big[x=\cos\theta_3\Big] = 4\pi\int\limits_0^\infty k^3dk \int\limits_{-1}^1 dx\,\sqrt{1-x^2}. \qquad (35)$$

Taking into account, that $k^\mu p_\mu = kp\cos\theta_3 = kpx$, the integral can be written as

$$\begin{aligned}
&\int d^4k \frac{1}{k^2(k+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)} =\\
&\qquad = 4\pi\int\limits_0^\infty k\,dk \int\limits_{-1}^1 dx\, \frac{\sqrt{1-x^2}}{(k^2+2kpx+p^2)\Big(1+k^{2n}/\Lambda^{2n}\Big)} =\\
&\qquad\qquad = 2\pi\int\limits_0^\infty k\,dk \oint\limits_C dx\, \frac{\sqrt{1-x^2}}{(k^2+2kpx+p^2)\Big(1+k^{2n}/\Lambda^{2n}\Big)},
\end{aligned} \qquad (36)$$

where the contour C is presented at Figure 3. The integrand here has singularities at branch points $x=\pm 1$, a pole $x=\infty$ and a pole

$$x_0 = -\frac{k^2+p^2}{2kp}. \qquad (37)$$

Then it is easy to see, that

$$\oint\limits_C dx\,\frac{\sqrt{1-x^2}}{k^2+2kpx+p^2}=2\pi i\,\mathrm{Res}\bigg(\frac{\sqrt{1-x^2}}{k^2+2kpx+p^2},\,x=\infty\bigg)-$$
$$-2\pi i\,\mathrm{Res}\bigg(\frac{\sqrt{1-x^2}}{k^2+2kpx+p^2},\,x=x_0\bigg)=2\pi i\bigg(-i\frac{k^2+p^2}{4k^2p^2}+i\frac{|k^2-p^2|}{4k^2p^2}\bigg). \qquad (38)$$

Therefore, the integral over angles is reduced to

$$\oint dx\,\frac{\sqrt{1-x^2}}{k^2+2kpx+p^2}=\begin{cases}\dfrac{\pi}{k^2}, & k\ge p;\\[2ex] \dfrac{\pi}{p^2}, & p\ge k\end{cases} \qquad (39)$$

and finally

$$I_1=2\pi^2\int\limits_0^p dk\,\frac{k}{p^2}\,\frac{1}{\Big(1+k^{2n}/\Lambda^{2n}\Big)}+2\pi^2\int\limits_p^\infty dk\,\frac{1}{k}\,\frac{1}{\Big(1+k^{2n}/\Lambda^{2n}\Big)}=$$
$$=\pi^2+o(1)+\frac{\pi^2}{n}\ln\frac{\Lambda^{2n}+p^{2n}}{p^{2n}}=2\pi^2\Big(\ln\frac{\Lambda}{p}+\frac{1}{2}\Big)+o(1). \qquad (40)$$

Integral I_2 can be calculated using standard methods [42]. First, using an identity

$$\frac{1}{ab}=\int\limits_0^1 dy\,\frac{1}{\Big(ay+b(1-y)\Big)^2}, \qquad (41)$$

it can be written as

$$I_2=\int\limits_0^1 dy\int d^4k\,\Bigg(\frac{1}{\Big(k^2+2kpy+yp^2\Big)^2}-\frac{1}{\Big(k^2+2kpy+yp^2+M^2\Big)^2}\Bigg). \qquad (42)$$

Each of this integrals diverges, but their difference is finite. Therefore, to simplify calculations it is convenient to use an auxiliary regularization, for example, the dimensional regularization. Then the integrals in equation (42) can be easily taken:

$$I_2=$$
$$=\lim_{D\to4}\pi^2\int\limits_0^1 dy\,\frac{\Gamma(2-D/2)}{\Gamma(2)}\bigg(\Big(y(1-y)\,p^2\Big)^{D/2-2}-\Big(y(1-y)\,p^2+M^2\Big)^{D/2-2}\bigg)=$$

$$= \pi^2 \int_0^1 dy\, \ln\left(1 + \frac{M^2}{y(1-y)\,p^2}\right) = \tag{43}$$

$$= 2\pi^2\left(\ln\frac{M}{p} + \sqrt{1+\frac{4M^2}{p^2}}\mathrm{arctanh}\sqrt{\frac{p^2}{4M^2+p^2}}\right).$$

To calculate divergent part of integral I_3 note, that $I_3 = I_3(p/\Lambda)$ and due to the logarithmical divergence

$$I_3 = a_1 \ln^2\frac{\Lambda}{p} + a_2 \ln\frac{\Lambda}{p} + \sum_{i=0}^{\infty} b_i \left(\frac{p^2}{\Lambda^2}\right)^i. \tag{44}$$

If $a_1 = 0$, then it is possible to find

$$a_2 = \lim_{p\to 0}\frac{dI_3}{d\ln\Lambda} = \int d^4k\, d^4q\, \frac{4n\, k^{2n-2}}{\Lambda^{2n}\Big(1 + k^{2n}/\Lambda^{2n}\Big)^2}\, \frac{q^2 + k_\mu q_\mu}{(k+q)^4 q^4}. \tag{45}$$

If this limit does not exist, then $a_1 \ne 0$. The integral in the right hand side of equation (45) can be taken, using four-dimensional spherical coordinates:

$$\int d^4q\, \frac{q^2 + k_\mu q_\mu}{(k+q)^4 q^4} = 4\pi \int_0^\infty dq \int_{-1}^1 dx\, \frac{(q+kx)\sqrt{1-x^2}}{(k^2+2kqx+q^2)^2} = \tag{46}$$

$$= -2\pi \int_{-1}^1 dx \int_0^\infty dq\, \frac{d}{dq}\frac{\sqrt{1-x^2}}{(k^2+2kqx+q^2)} = \frac{2\pi}{k^2}\int_{-1}^1 dx\, \sqrt{1-x^2} = \frac{\pi^2}{k^2},$$

so that

$$a_2 = 4n\pi^2 \int d^4k\, \frac{k^{2n-4}}{\Lambda^{2n}\Big(1 + k^{2n}/\Lambda^{2n}\Big)^2} = 4\pi^4, \tag{47}$$

Therefore, from equation (44) we conclude, that

$$I_3 = 4\pi^4 \ln\frac{\Lambda}{p} + O(1). \tag{48}$$

In order to calculate integral I_4 let us note, that

$$\int \frac{d^4k}{(2\pi)^4}\, \frac{M^2}{(k^2+M^2)^2\Big((k+p)^2 + M^2\Big)} = f(p/M). \tag{49}$$

Therefore, instead of taking the limit $M\to\infty$ it is possible to take the limit $p \to 0$, so that

$$I_4 = \int d^4k\, \frac{M^2}{(k^2+M^2)^3} = \frac{\pi^2}{2}. \tag{50}$$

Divergent part of integral I_5 can be also easily calculated, because

$$I_5 = I_1^2 = 4\pi^4\left(\ln^2\frac{\Lambda}{p} + \ln\frac{\Lambda}{p}\right) + O(1). \qquad (51)$$

To find a divergent part of I_6 let us consider

$$\begin{aligned}
&\lim_{p\to 0}\Lambda\frac{d}{d\Lambda}\Big(I_5 - I_6\Big) = \\
&= \lim_{p\to 0}\Lambda\frac{d}{d\Lambda}\int d^4k\, d^4q\left(1 - \frac{(k+q+2p)^2}{(k+q+p)^2}\right)\times \\
&\qquad\qquad\times\frac{1}{k^2(k+p)^2q^2(q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)} = \\
&= \lim_{p\to 0}\int d^4k\, d^4q\,\frac{-2(k+q)p - 3p^2}{(k+q+p)^2}\times \\
&\qquad\qquad\times\frac{4nq^{2n}/\Lambda^{2n}}{k^2(k+p)^2q^2(q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)^2} = 0. \qquad (52)
\end{aligned}$$

(It is important to note, that all integrals here are convergent.) Therefore,

$$I_6 = I_1^2 + O(1) = 4\pi^4\left(\ln^2\frac{\Lambda}{p} + \ln\frac{\Lambda}{p}\right) + O(1). \qquad (53)$$

A divergent part of I_7 can be calculated similarly:

$$\begin{aligned}
&\lim_{p\to 0}\Lambda\frac{d}{d\Lambda}\Big(I_5 - 2I_7\Big) = \\
&= \lim_{p\to 0}\Lambda\frac{d}{d\Lambda}\int d^4k\, d^4q\left(\frac{1}{(k+p)^2} - \frac{2}{(k+q+p)^2}\right)\times \\
&\qquad\qquad\times\frac{1}{k^2q^2(q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)} = \\
&= \lim_{p\to 0}\Lambda\frac{d}{d\Lambda}\int d^4k\, d^4q\,\frac{q^2 + 2(k+p)q - (k+p)^2}{(k+p)^2(k+q+p)^2}\times \\
&\qquad\qquad\times\frac{1}{k^2q^2(q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)} = \\
&= \lim_{p\to 0}\Lambda\frac{d}{d\Lambda}\int d^4k\, d^4q\,\frac{q^2 + 2(k+p)q - (q+p)^2}{(k+p)^2(k+q+p)^2}\times \\
&\qquad\qquad\times\frac{1}{k^2q^2(q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)} = \\
&= \lim_{p\to 0}\Lambda\frac{d}{d\Lambda}\int d^4k\, d^4q\,\frac{2kq - p^2}{(k+p)^2(k+q+p)^2}\times
\end{aligned}$$

$$\times \frac{1}{k^2q^2(q+p)^2\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)} =$$

$$= \int d^4k\, d^4q\, \frac{2kq}{k^4q^4(k+q)^2}\,\Lambda\frac{d}{d\Lambda}\left[\frac{1}{\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)}\right] =$$

$$= 8n \int d^4q\, d^4k\, \frac{kq}{k^4q^4(k+q)^2}\frac{q^{2n}/\Lambda^{2n}}{\Big(1+k^{2n}/\Lambda^{2n}\Big)\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}. \qquad (54)$$

To calculate this integral we again use four-dimensional spherical coordinates and direct fourth axis along q^μ. Then similar to the case, considered above, the integral over angles is reduced to

$$4\pi \int\limits_{-1}^{1} dx\, \frac{x\sqrt{1-x^2}}{k^2+2kqx+q^2} = 2\pi \oint\limits_{C} dx\, \frac{x\sqrt{1-x^2}}{k^2+2kqx+q^2} =$$

$$= 4\pi^2 i\, \mathrm{Res}\left(\frac{x\sqrt{1-x^2}}{k^2+2kqx+q^2},\, x=\infty\right) - 4\pi^2 i\, \mathrm{Res}\left(\frac{x\sqrt{1-x^2}}{k^2+2kqx+q^2},\, x=x_0\right) =$$

$$= 4\pi^2 i\left(-\frac{i}{4kq} + \frac{i(k^2+q^2)^2}{8k^3q^3} - \frac{i|k^2-q^2|(k^2+q^2)}{8k^3q^3}\right), \qquad (55)$$

so that

$$2\pi \oint dx\, \frac{x\sqrt{1-x^2}}{k^2+2kqx+q^2} = \begin{cases} -\dfrac{\pi^2 q}{k^3}, & k \geq q; \\[2ex] -\dfrac{\pi^2 k}{q^3}, & q \geq k. \end{cases} \qquad (56)$$

Therefore,

$$\lim_{p\to 0} \Lambda\frac{d}{d\Lambda}\Big(I_5 - 2I_7\Big) = -16n\,\pi^4 \int\limits_0^\infty dq\, \frac{q^{2n}/\Lambda^{2n}}{\Big(1+q^{2n}/\Lambda^{2n}\Big)^2} \times$$

$$\times\left(\int\limits_q^\infty dk\, \frac{q}{k^3\Big(1+k^{2n}/\Lambda^{2n}\Big)} + \int\limits_0^q dk\, \frac{k}{q^3\Big(1+k^{2n}/\Lambda^{2n}\Big)}\right) =$$

$$= -4n\,\pi^4 \int\limits_0^\infty dx\, \frac{x^n}{(1+x^n)^2} \int\limits_0^{1/x} \frac{dy}{1+y^{-n}} - 4n\,\pi^4 \int\limits_0^\infty dx\, \frac{x^n}{(1+x^n)^2} \int\limits_0^{1/x} \frac{dy}{1+y^n} =$$

$$= -4n\,\pi^4 \int\limits_0^\infty dx\, \frac{x^{n-1}}{(1+x^n)^2} = -4\pi^4 \qquad (57)$$

and finally

$$I_7 = \frac{1}{2} I_1^2 + 2\pi^4 \ln\frac{\Lambda}{p} + O(1) = 2\pi^4 \Big(\ln^2\frac{\Lambda}{p} + 2\ln\frac{\Lambda}{p} \Big) + O(1). \tag{58}$$

Using equation (43) integral I_8 can be written as

$$I_8 = 2\pi^2 \int d^4q \, \frac{1}{q^2(q+p)^2\Big(1+q^{2n}/\Lambda^{2n}\Big)^2} \Big(\ln\frac{M}{q} + \sqrt{1+\frac{4M^2}{q^2}} \mathrm{arctanh} \sqrt{\frac{q^2}{4M^2+q^2}} \Big), \tag{59}$$

where $M = a\Lambda$, a being a constant. To calculate the divergent part of this integral let us consider first an integral

$$I_f \equiv \int d^4q \frac{1}{q^2(q+p)^2} f\Big(\Lambda/q\Big) = I_f\Big(\Lambda/p\Big), \tag{60}$$

where f is a function. Differentiating I_f over $\ln\Lambda$ and setting then $p=0$, we obtain, that

$$\Lambda \frac{dI_f}{d\Lambda}\Big|_{p=0} = \int d^4q \frac{1}{q^4} \Lambda \frac{d}{d\Lambda} f\Big(\Lambda/q\Big) = -\int d^4q \frac{1}{q^3} \frac{d}{dq} f\Big(\Lambda/q\Big) =$$

$$= -2\pi^2 \int\limits_0^\infty dq \, \frac{d}{dq} f\Big(\Lambda/q\Big) = 2\pi^2 \Big(f(\infty) - f(0) \Big). \tag{61}$$

So, if the values $f(\infty)$ and $f(0)$ are finite, then

$$I_f = 2\pi^2 \Big(f(\infty) - f(0) \Big) \ln\frac{\Lambda}{p} + O(1). \tag{62}$$

If the function f is taken to be

$$f\Big(\Lambda/q\Big) = \frac{2\pi^2}{\Big(1+q^{2n}/\Lambda^{2n}\Big)^2} \sqrt{1+\frac{4M^2}{q^2}} \, \mathrm{arctanh} \sqrt{\frac{q^2}{4M^2+q^2}}, \tag{63}$$

then from equation (62) we conclude, that

$$2\pi^2 \int d^4q \frac{1}{q^2(q+p)^2\Big(1+q^{2n}/\Lambda^{2n}\Big)^2} \sqrt{1+\frac{4M^2}{q^2}} \mathrm{arctanh} \sqrt{\frac{q^2}{4M^2+q^2}} =$$

$$= 4\pi^4 \ln\frac{\Lambda}{p} + O(1). \tag{64}$$

However, it is impossible to substitute in equation (62)

$$f\Big(\Lambda/q\Big) = \frac{2\pi^2}{\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}\ln\frac{M}{q} \tag{65}$$

because $f(\infty)$ does not exist. Nevertheless, the function f can chosen in following form:

$$\begin{aligned} f\Big(\Lambda/q\Big) &= \Lambda\frac{d}{d\Lambda}\left(\frac{2\pi^2}{\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}\ln\frac{M}{q}\right) = \\ &= \frac{8\pi^2 n\, q^{2n}/\Lambda^{2n}}{\Big(1+q^{2n}/\Lambda^{2n}\Big)^3}\ln\frac{M}{q} + \frac{2\pi^2}{\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}, \end{aligned} \tag{66}$$

so that $f(0)=0$ and $f(\infty)=2\pi^2$. Then from equation (61) we obtain, that

$$\left(\Lambda\frac{d}{d\Lambda}\right)^2 2\pi^2\int d^4q\frac{1}{q^2(q+p)^2\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}\ln\frac{M}{q} = 4\pi^4\ln\frac{\Lambda}{p} + O(1) \tag{67}$$

and, therefore,

$$2\pi^2\int d^4q\frac{1}{q^2(q+p)^2\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}\ln\frac{M}{q} = 2\pi^4\ln^2\frac{\Lambda}{p} + O\Big(\ln\frac{\Lambda}{p}\Big). \tag{68}$$

Then it is necessary to calculate logarithmical divergences. For this purpose we subtract from integral (68) terms, proportional $\ln^2\Lambda/p$ and differentiate the result over $\ln\Lambda$:

$$\begin{aligned} &\lim_{p\to 0}\Lambda\frac{d}{d\Lambda}\left[2\pi^2\int d^4q\frac{1}{q^2(q+p)^2\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}\ln\frac{M}{q} - 2\pi^4\ln^2\frac{\Lambda}{p}\right] = \\ &= \lim_{p\to 0}\left\{-2\pi^2\int d^4q\frac{1}{q^2(q+p)^2}\,q\frac{d}{dq}\left(\frac{1}{\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}\ln\frac{M}{q}\right) - 4\pi^4\ln\frac{\Lambda}{p}\right\} = \\ &= \lim_{p\to 0}\left\{-4\pi^4\int\limits_0^p dq\,\frac{q^2}{p^2}\,\frac{d}{dq}\left(\frac{1}{\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}\ln\frac{M}{q}\right) - \right. \\ &\qquad \left. -4\pi^4\int\limits_p^\infty dq\,\frac{d}{dq}\left(\frac{1}{\Big(1+q^{2n}/\Lambda^{2n}\Big)^2}\ln\frac{M}{q}\right) - 4\pi^4\ln\frac{\Lambda}{p}\right\} = 2\pi^4 + 4\pi^4\ln\frac{M}{\Lambda}. \end{aligned} \tag{69}$$

From equations (59), (64), (68) and (69), we see, that the divergent part of I_8 is equal to

$$I_8 = 2\pi^4\left(\ln^2\frac{\Lambda}{p} + 2\ln\frac{\Lambda}{p}\Big(\ln\frac{M}{\Lambda}+\frac{3}{2}\Big)\right) + O(1). \tag{70}$$

Therefore, integrals (33) can be finally written as

$$\begin{aligned}
I_1 &= 2\pi^2\Big(\ln\frac{\Lambda}{p}+\frac{1}{2}\Big)+o(1);\\
I_2 &= 2\pi^2\Big(\ln\frac{M}{p}+\sqrt{1+\frac{4M^2}{p^2}}\,\mathrm{arctanh}\sqrt{\frac{p^2}{4M^2+p^2}}\Big);\\
I_3 &= 4\pi^4\ln\frac{\Lambda}{p}+O(1);\\
I_4 &= \frac{\pi^2}{2};\\
I_5 &= 4\pi^4\Big(\ln^2\frac{\Lambda}{p}+\ln\frac{\Lambda}{p}\Big)+O(1);\\
I_6 &= 4\pi^4\Big(\ln^2\frac{\Lambda}{p}+\ln\frac{\Lambda}{p}\Big)+O(1);\\
I_7 &= 2\pi^4\Big(\ln^2\frac{\Lambda}{p}+2\ln\frac{\Lambda}{p}\Big)+O(1);\\
I_8 &= 2\pi^4\Big(\ln^2\frac{\Lambda}{p}+2\ln\frac{\Lambda}{p}\Big(\ln\frac{M}{\Lambda}+\frac{3}{2}\Big)\Big)+O(1).
\end{aligned} \tag{71}$$

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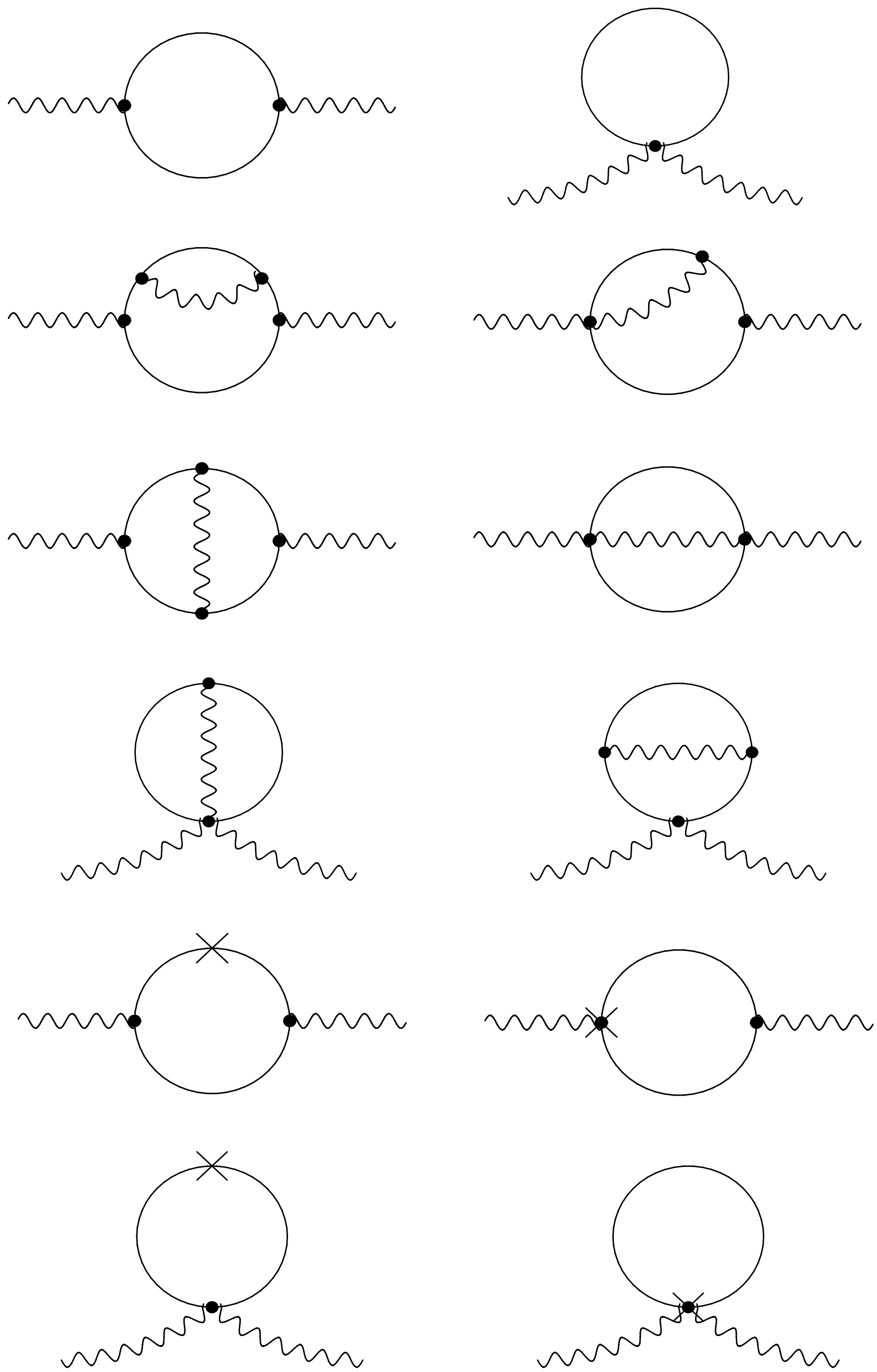

Figure 1: Feinman graphs, giving nontrivial contributions to the two-loop β-function of $N=1$ supersymmetric electrodynamics.

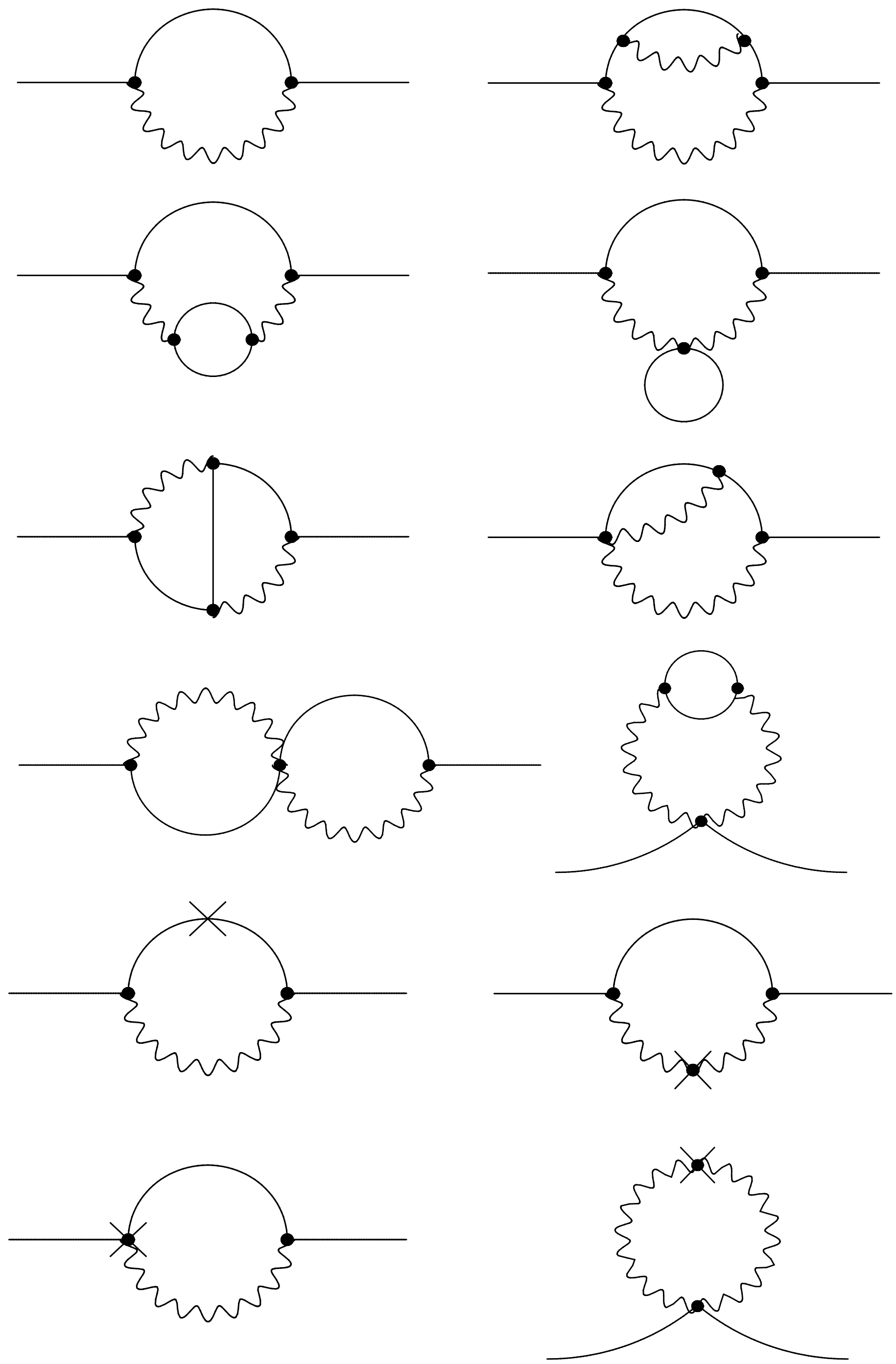

Figure 2: Feinman graphs, giving nontrivial contributions to the two-loop anomalous dimension of $N = 1$ supersymmetric electrodynamics.

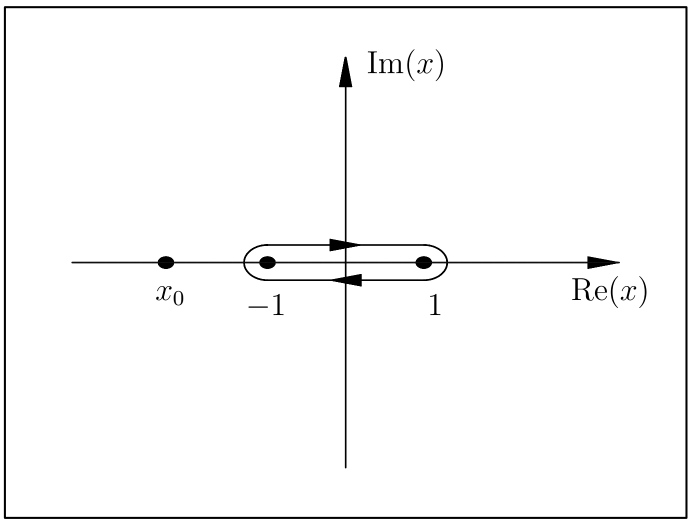

Figure 3: Contour C for calculation of integral over x.